Percolation and the Large Scale Structure of the Universe
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- Understanding the formation of structure in the Universe
- Precision cosmology vs. qualitative relationships
- New tools for theory and observations
- Connections to statistical mechanics?
- MNRAS '06 and in preparation

![Graph showing dark matter simulation using MC²](image)

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Interpreting Cosmic Maps: **Statistics ↔ Dynamics**

**Two-point Statistics**
Relatively robust, “easy” to compute and compare to observations. Clean theoretical interpretation.

**Shape/Topology Indicators**
Useful as characterization tools, but connection to underlying gravitational physics remains unclear.

**Higher-point statistics**
Tedious to compute, theoretical interpretations not so straightforward. Hard to measure.

**Phenomenology**
Halo models useful as statistical descriptors and to provide basic intuition, but connection to underlying theory somewhat indirect.

Homogeneity scale ~ 100 Mpc
SDSS Main Sample ~ 400 Mpc

Network/Void duality
I. Singularities in Lagrangian Space

Singularity structure of local map approximations:

$$\vec{x}(\vec{q}, t) = \vec{q} + D(t)\vec{s}_R(\vec{q})$$

$$d_{ik} = \frac{\partial s_i}{\partial q_k}$$

II. Cosmic Web

“Correlation bridges” from considering conditional multi-point correlation functions (e.g., of the primordial shear field)

II. Structural “Building Blocks”

Although the basic units of structure may be so indentified, we desire a global, quantitative measure of network structure.
I. Continuous Structural Transition
As a function of some control parameter, a physical property changes continuously near a singular point.

II. Percolation I
Percolation = probabilistic models with continuous (percolation) transition

III. Percolation II
No concepts from equilibrium statistical mechanics or the existence of Hamiltonians required to study percolation.

IV. Universality/Scaling
Near the transition point, percolation properties should split up into a small number of universality classes (e.g., morphology of percolating cluster).

Simple scaling laws expected near the transition --
I. Continuum Models
Instead of lattice-based models, consider continuous random fields, control parameter being amplitude or density, etc. Dual models map to random networks.

II. Gaussian Random Fields
A popular, very simple, class of continuum model; no exact results available for percolation properties!

III. Scaling Ansatz (single-variable)

\[ n_s(p) = s^{-\tau} f[(p - p_c)s^\sigma], \quad (p \to p_c, \ s \to \infty) \]

Normalized cluster number (per lattice site) of a given size, as a function of on-site occupation probability, near the percolation transition, and for large sizes.

Simple (continuum) versions of this ansatz provide very good fits to numerical results.

Basis of application to cosmology --
I. Gaussian Fields
Uniquely specified by their two-point statistics (power spectra).

II. Symmetry
Exact symmetry between overdense and underdense excursion sets.

III. Percolation Ansatz
\[ FF_1 = A(FF - FF_c)^\nu \]

\( FF_1 \) is the filling factor of the percolating region. \( FF_c \) is the filling factor when percolation occurs. The ansatz applies when \( FF > FF_c \).

Percolation threshold decreases with increase of large-scale power.
**Percolation with Particles: Forward Map**

**Initial Percolating Region**
Particles pretty much track the density, overall large-scale morphology maintained by evolution.

**Final Percolating Region**
Particles map to final percolating region, but they don’t percolate -- initial region is compressed and fragmented.

*340 Mpc/h*
Nature or Nurture?

1. “Percolating” Particles
   All particles in a percolating region (not equivalent to density cut!)

II. Forward/Inverse Maps
   Particle from initial percolating region(s) are mapped to final percolating regions. But these particles do not themselves form a percolating cluster: they fragment into a very large number of isolated regions (overdense regions collapse), a compression factor of more than an order of magnitude.

Inverse particle map percolates --
LCDM Percolation Transition at $z=0$

**I. Broken Symmetry**
Symmetry between underdense and overdense excursions is broken by gravitational evolution.

**II. Percolation**
Ansatz still holds separately for the under and overdense sets. Overdense set percolates much more easily (more large-scale power), underdense percolation set goes the other way: Nonlinear evolution amplifies the network structure present in the cosmic web.
Summary and Outlook

I. Percolation statistics can now be calculated robustly and accurately for cosmological density fields (in simulations).

II. Percolation provides a useful global measure of the nature of cosmological structure, how much is controlled by the initial condition, and how much by gravitational evolution.

III. Percolation analysis is applicable to large-volume galaxy surveys (2-D/3-D). How hard to do vs. power spectrum or the two-point function? Explore systematics with mock catalogs.

IV. In statistical mechanics, percolation scaling laws have been predicted using RG methods. Can this -- or some other approach -- be an alternative to conventional perturbation theory to describe the gravitational instability vis a vis percolation?

V. Can particle percolation statistics be connected to phenomenological approaches to structure formation, such as the halo model?