Percolation and the Large Scale Structure of the Universe

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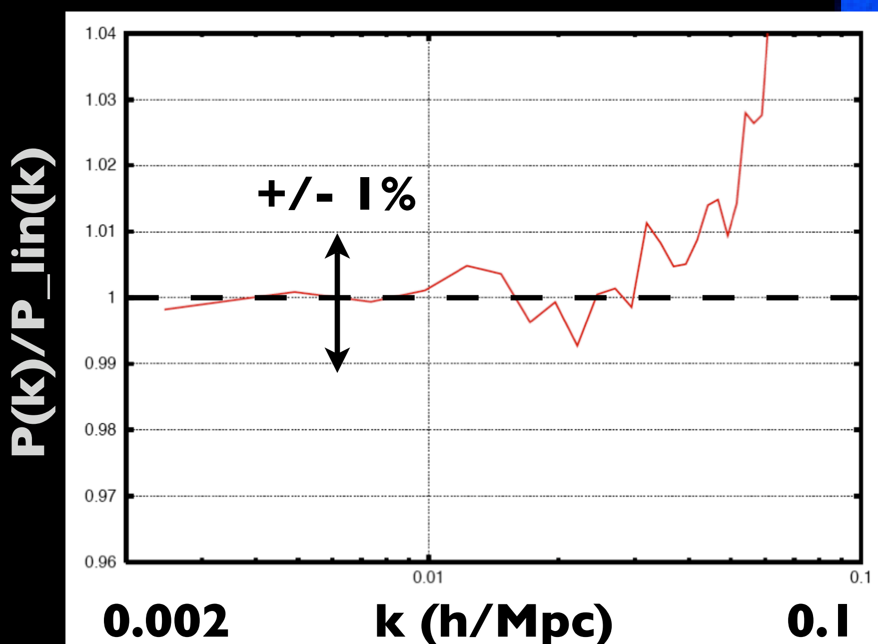
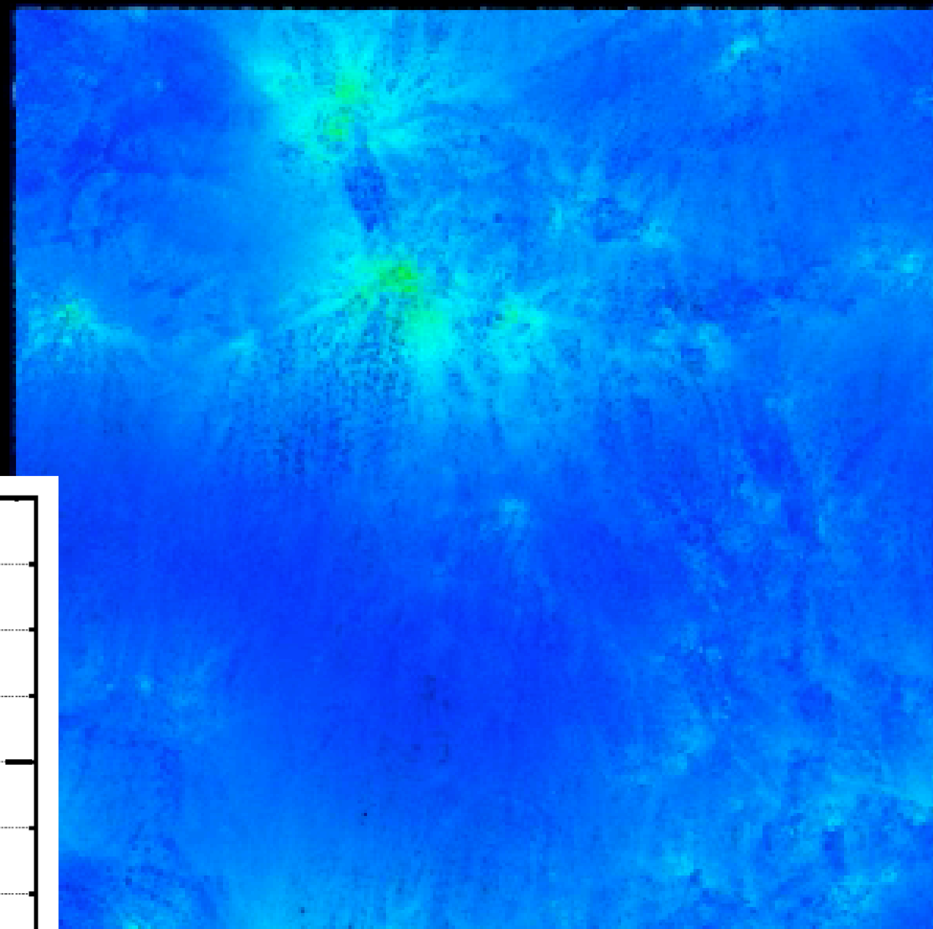
- Understanding the formation of structure in the Universe
- Precision cosmology vs. qualitative relationships
- New tools for theory and observations
- Connections to statistical mechanics?
- MNRAS '06 and in preparation





Dark matter simulation using MC^2

Interpreting Cosmic Maps: Statistics ⬌ Dynamics

Two-point Statistics

Relatively robust, "easy" to compute and compare to observations. Clean theoretical interpretation.

Shape/Topology Indicators

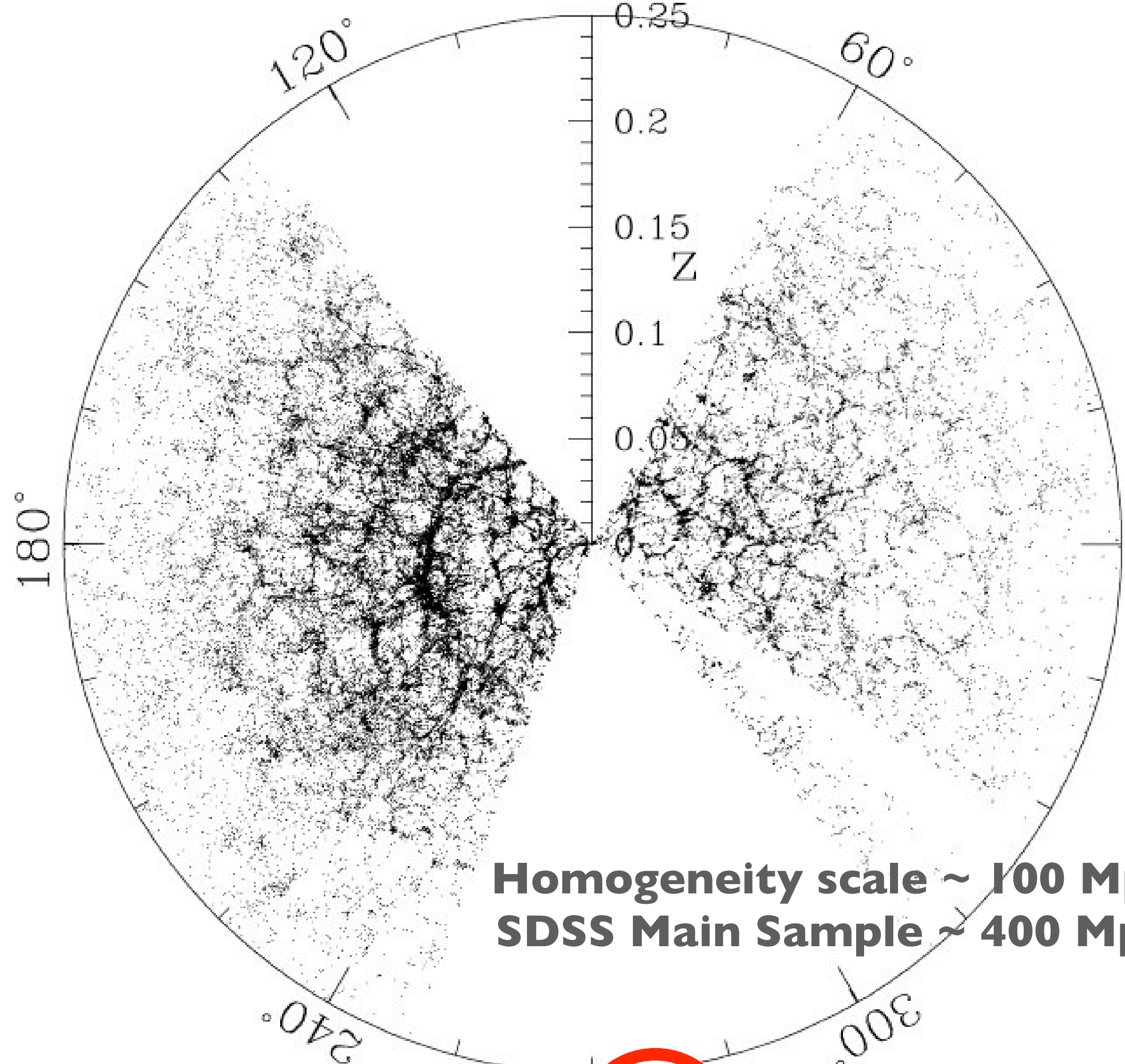
Useful as characterization tools, but connection to underlying gravitational physics remains unclear.

Higher-point statistics

Tedious to compute, theoretical interpretations not so straightforward. Hard to measure.

Phenomenology

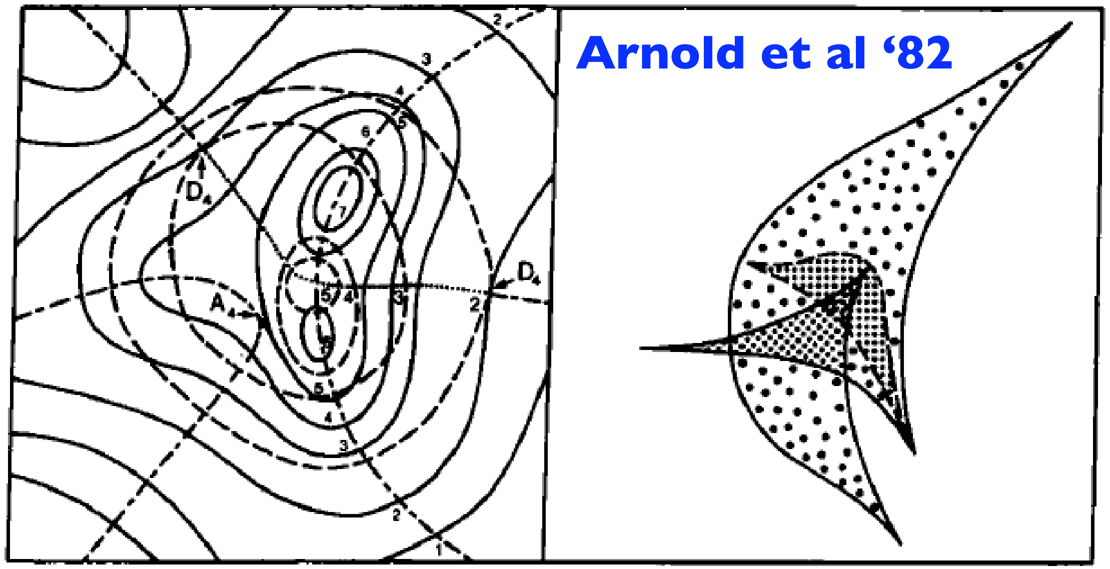
Halo models useful as statistical descriptors and to provide basic intuition, but connection to underlying theory somewhat indirect.

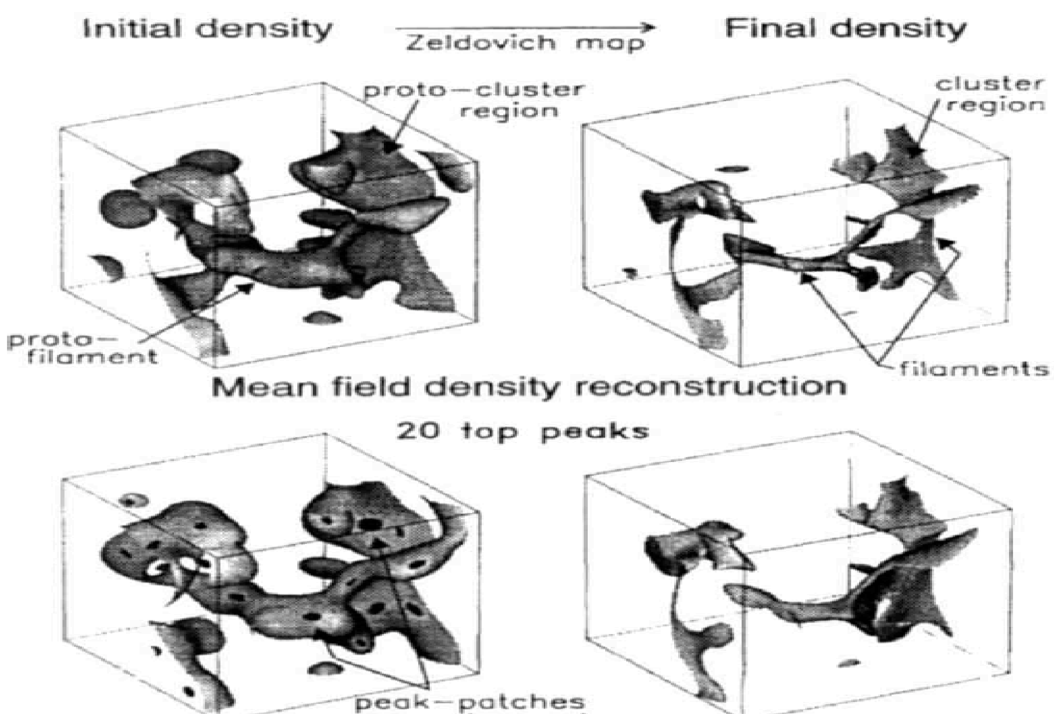


Homogeneity scale ~ 100 Mpc
SDSS Main Sample ~ 400 Mpc

Network/~~Void~~ duality

Local Descriptions of Structure Formation: Map-Dynamics Connection





Bond et al '96

I. Singularities in Lagrangian Space

Singularity structure of local map approximations:

$$\vec{x}(\vec{q},t) = \vec{q} + D(t)\vec{s}_R(\vec{q})$$

$$d_{ik} = \frac{\partial s_i}{\partial q_k}$$

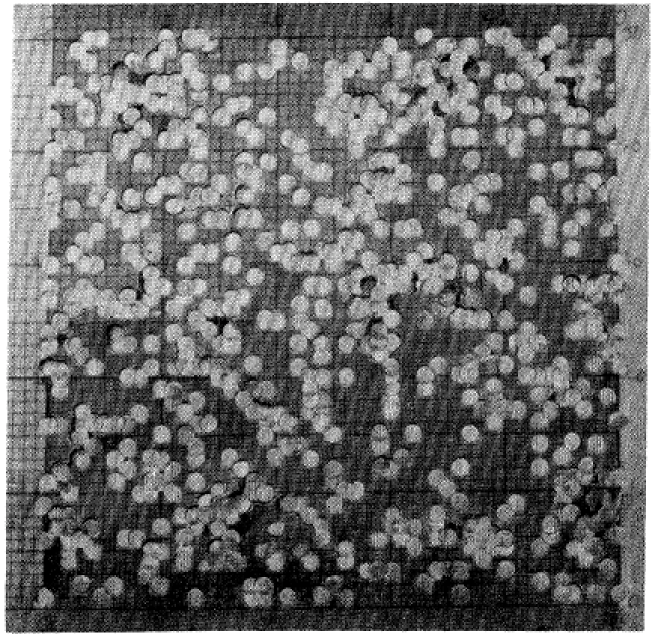
II. Cosmic Web

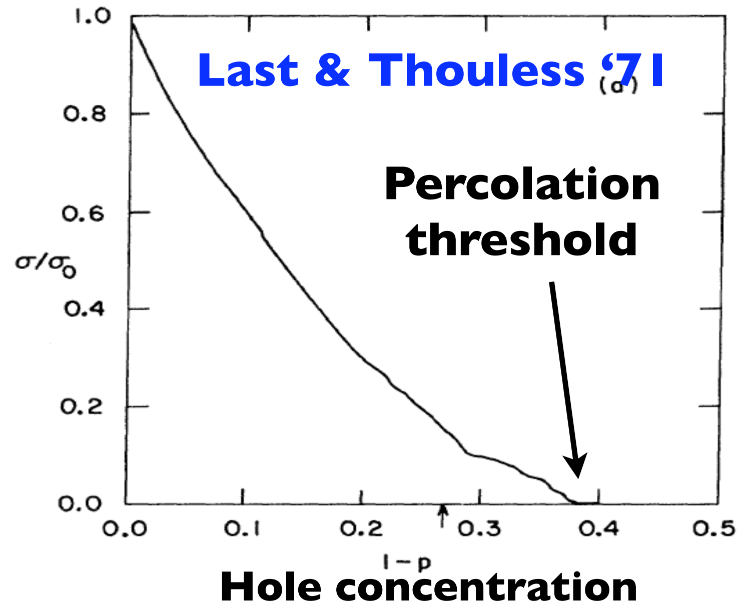
"Correlation bridges" from considering conditional multi-point correlation functions (e.g., of the primordial shear field)

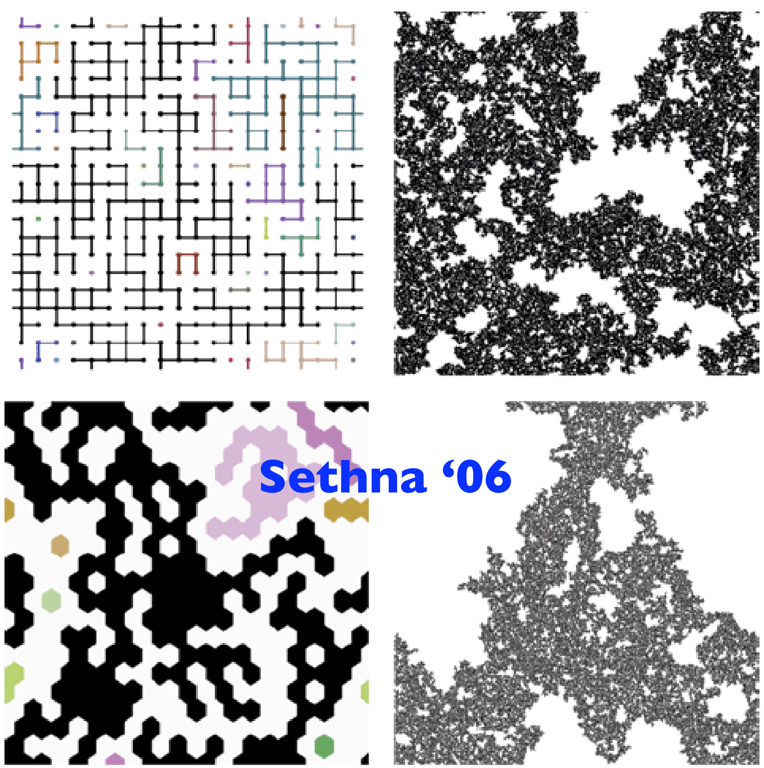
II. Structural "Building Blocks"

Although the basic units of structure may be so indentified, we desire a global, quantitative measure of network structure.

Percolation







I. Continuous Structural Transition

As a function of some control parameter, a physical property changes continuously near a singular point.

II. Percolation I

Percolation = probabilistic models with continuous (percolation) transition

III. Percolation II

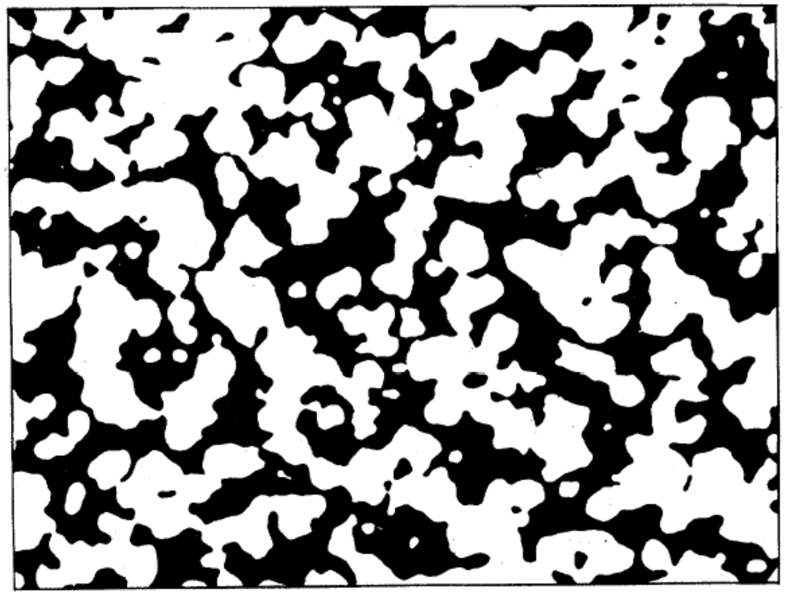
No concepts from equilibrium statistical mechanics or the existence of Hamiltonians required to study percolation.

IV. Universality/ Scaling

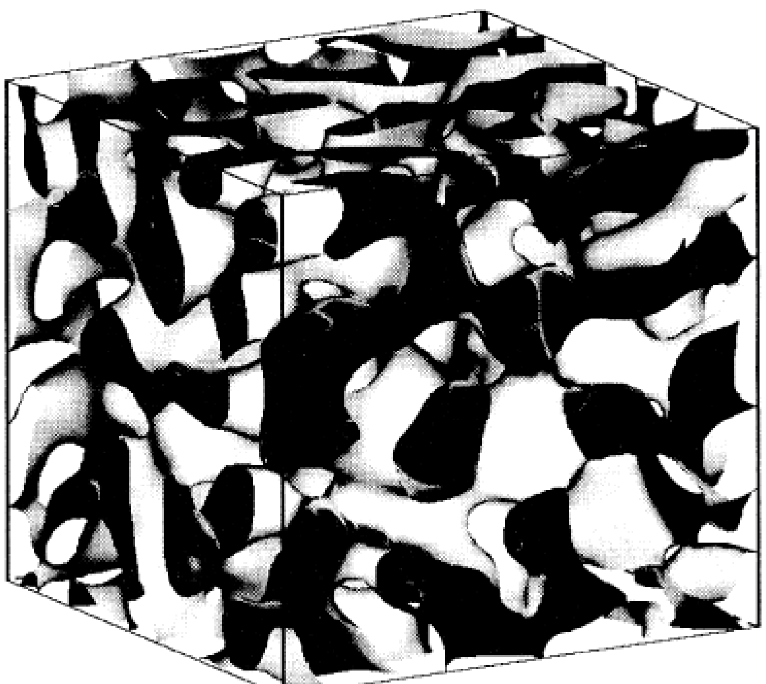
Near the transition point, percolation properties should split up into a small number of universality classes (e.g., morphology of percolating cluster).

Simple scaling laws expected near the transition --

Continuum Percolation



Smith & Lobb '79 (2-D)



Roberts & Teubner '95 (3-D)

I. Continuum Models

Instead of lattice-based models, consider continuous random fields, control parameter being amplitude or density, etc. Dual models map to random networks.

II. Gaussian Random Fields

A popular, very simple, class of continuum model; no exact results available for percolation properties!

III. Scaling Ansatz (single-variable)

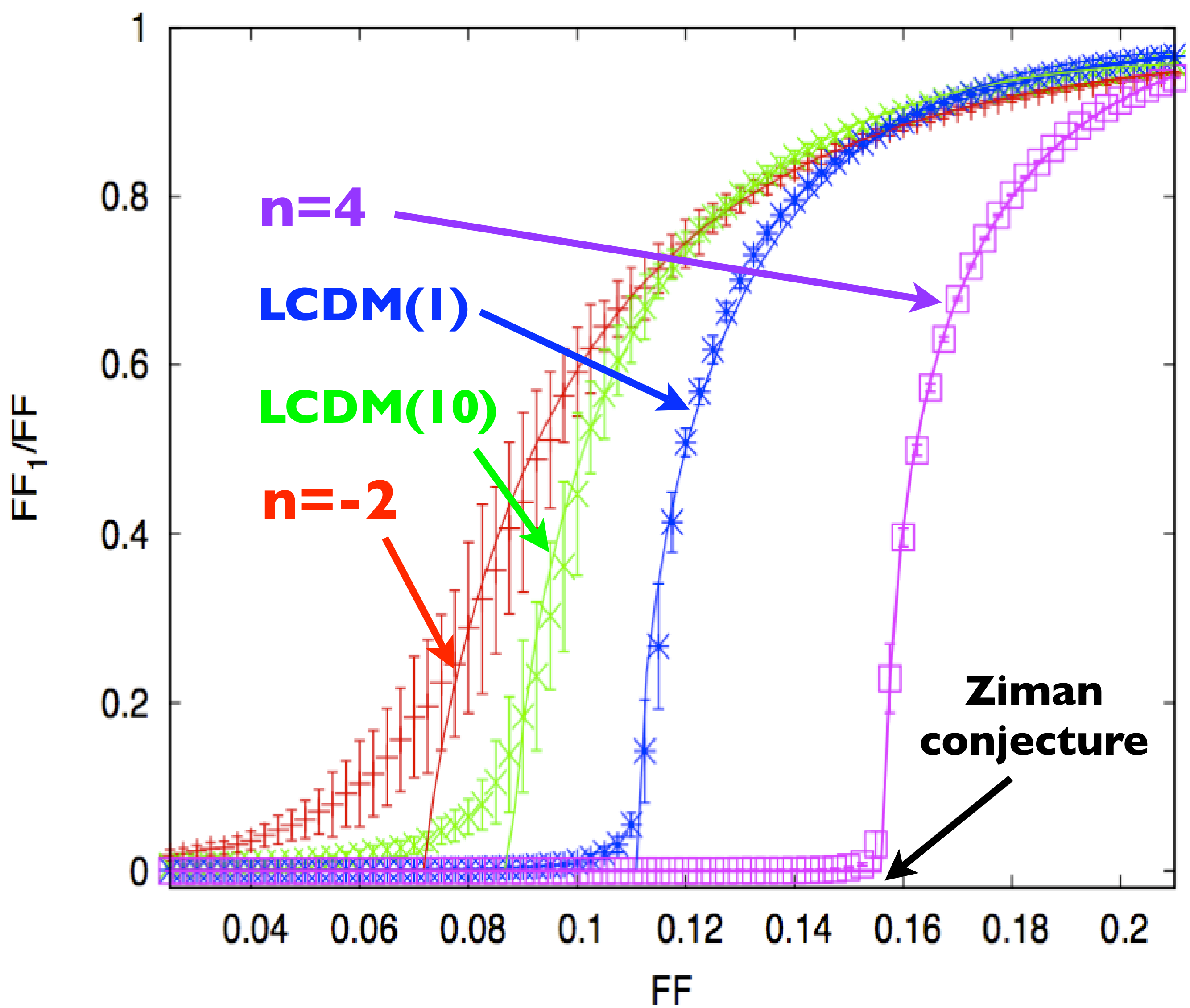
$$n_s(p) = s^{-\tau} f[(p - p_c)s^{\sigma}], \ (p \to p_c, \ s \to \infty)$$

Normalized cluster number (per lattice site) of a given size, as a function of on-site occupation probability, near the percolation transition, and for large sizes.

Simple (continuum) versions of this ansatz provide very good fits to numerical results.

Basis of application to cosmology --

Percolation in Gaussian Random Fields (Cosmological Initial Conditions)



Percolation threshold decreases with increase of large-scale power

I. Gaussian Fields

Uniquely specified by their two-point statistics (power spectra).

II. Symmetry

Exact symmetry between overdense and underdense excursion sets.

III. Percolation Ansatz

$$FF_1 = A(FF - FF_c)^{\nu}$$

FF_1 **is the filling factor of the percolating region.** FF_c **is the filling factor when percolation occurs. The ansatz applies when** $FF > FF_c$**.**

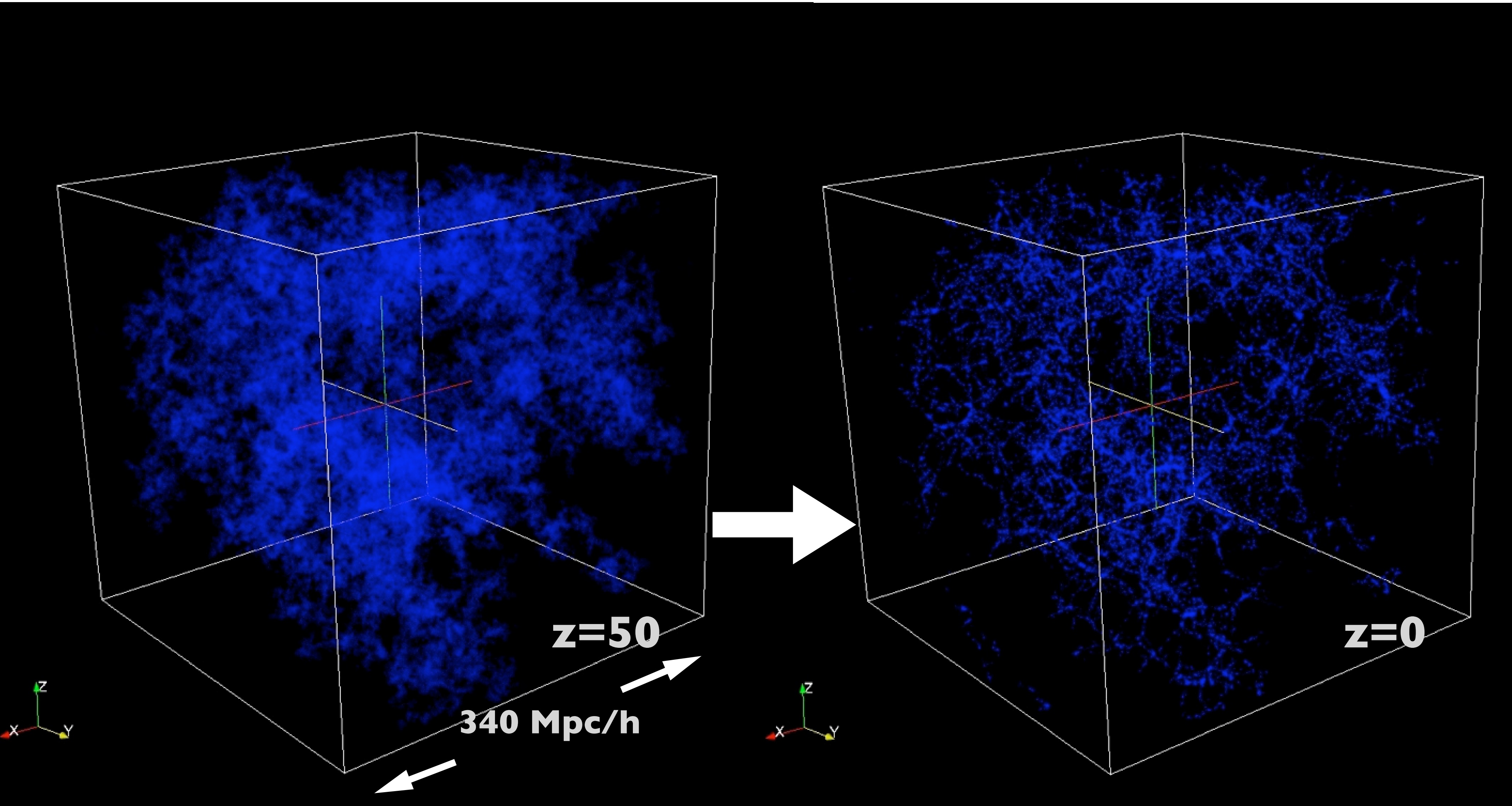
Percolation with Particles: Forward Map

Initial Percolating Region

Particles pretty much track the density, overall large-scale morphology maintained by evolution

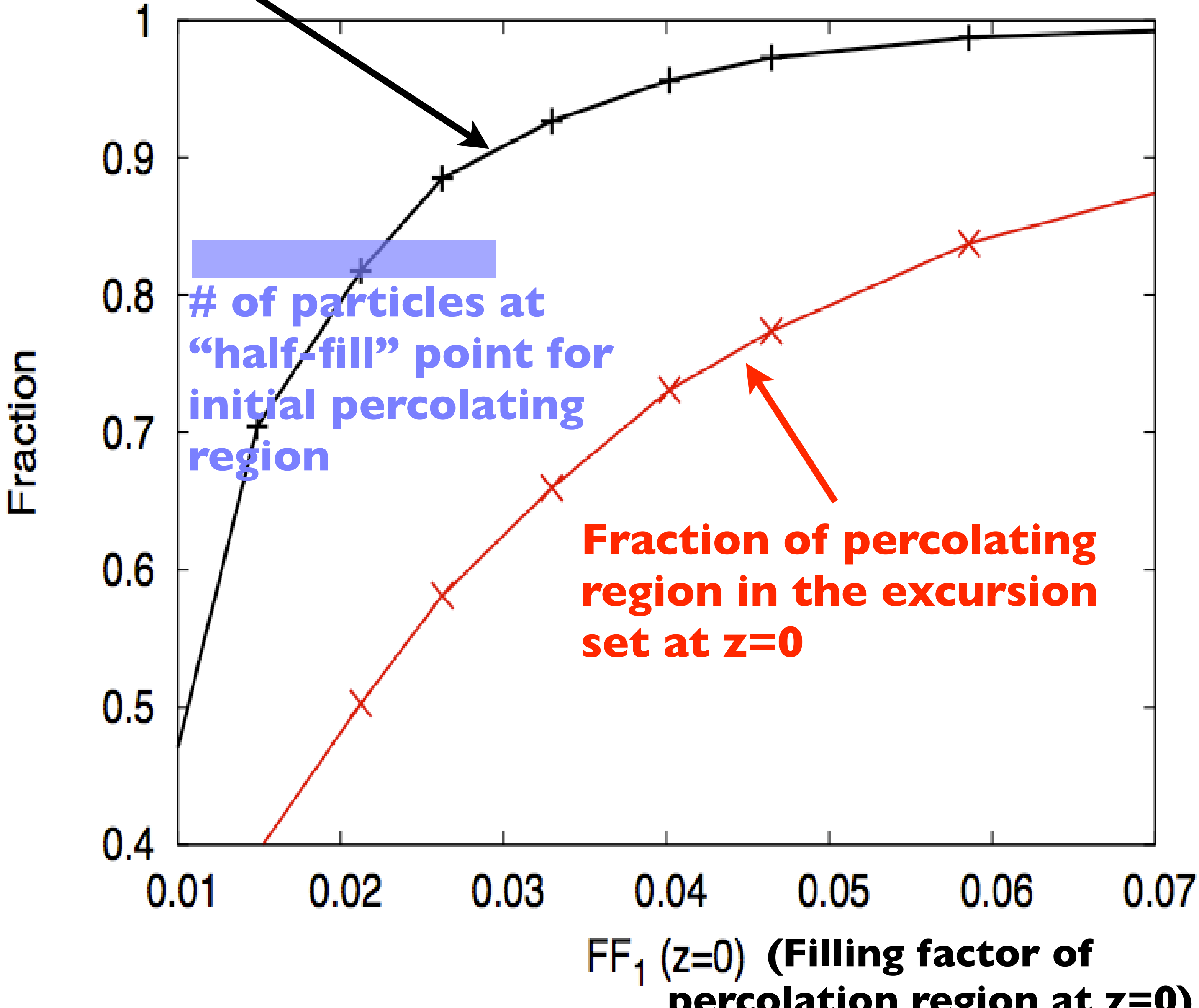
Final Percolating Region

Particles map to final percolating region, but they don't percolate -- initial region is compressed and fragmented



Nature or Nurture?

Fraction of particles from the initial percolating set, in the final percolating region at z=0



I. "Percolating" Particles

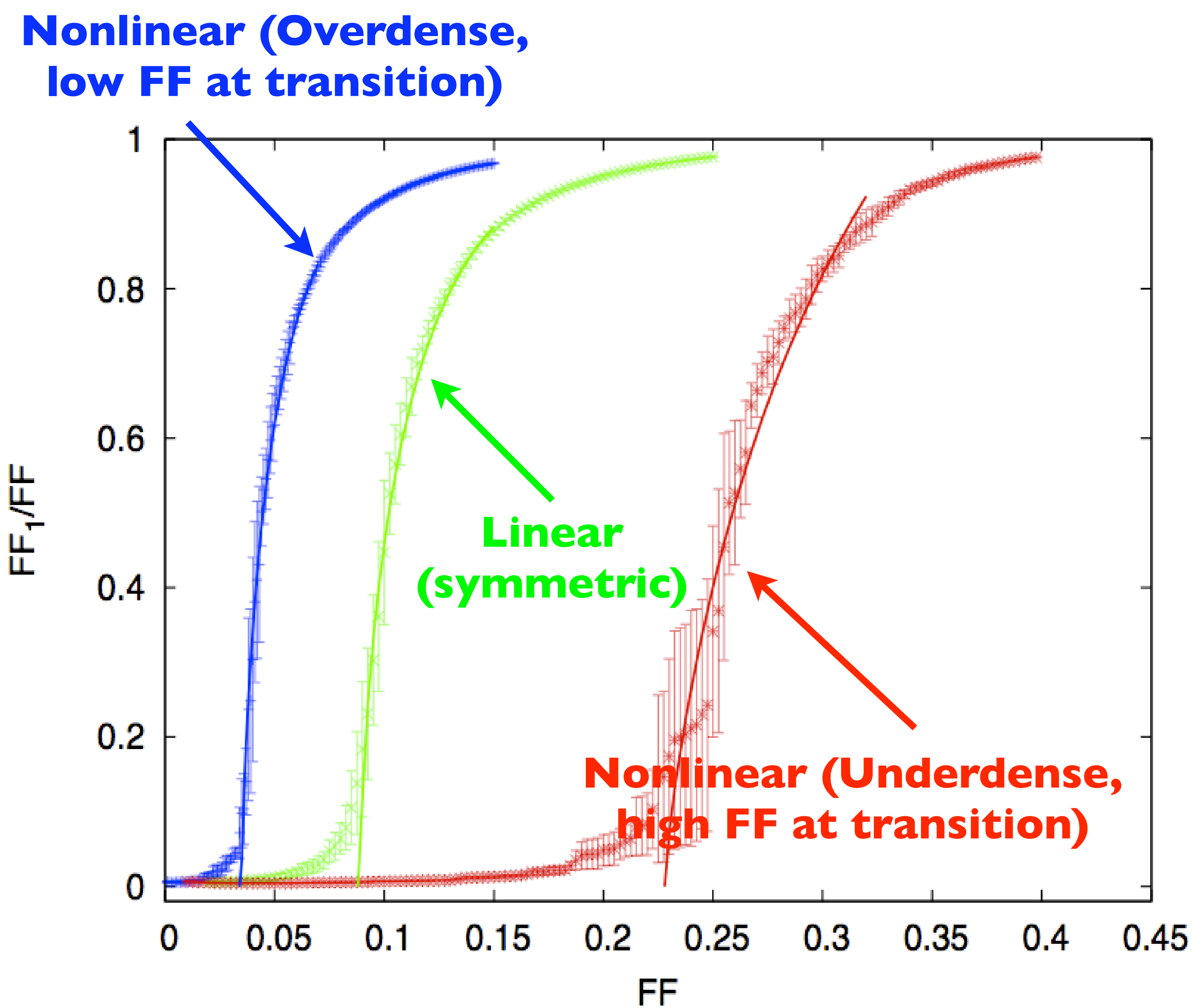
All particles in a percolating region (not equivalent to density cut!)

II. Forward/Inverse Maps

Particle from initial percolating region(s) are mapped to final percolating regions. But these particles do not themselves form a percolating cluster: they fragment into a very large number of isolated regions (overdense regions collapse), a compression factor of more than an order of magnitude.

Inverse particle map percolates --

LCDM Percolation Transition at z=0



I. Broken Symmetry

Symmetry between underdense and overdense excursions is broken by gravitational evolution

II. Percolation

Ansatz still holds separately for the under and overdense sets. Overdense set percolates much more easily (more large-scale power), underdense percolation set goes the other way: Nonlinear evolution amplifies the network structure present in the cosmic web.

Summary and Outlook

I. Percolation statistics can now be calculated robustly and accurately for cosmological density fields (in simulations).

II. Percolation provides a useful global measure of the nature of cosmological structure, how much is controlled by the initial condition, and how much by gravitational evolution.

III. Percolation analysis is applicable to large-volume galaxy surveys (2-D/3-D). How hard to do vs. power spectrum or the two-point function? Explore systematics with mock catalogs.

IV. In statistical mechanics, percolation scaling laws have been predicted using RG methods. Can this -- or some other approach -- be an alternative to conventional perturbation theory to describe the gravitational instability vis a vis percolation?

V. Can particle percolation statistics be connected to phenomenological approaches to structure formation, such as the halo model?